# Growth models and site index table of natural Korean pine forests

Sun Yuwen（孙玉文）  Li Shi（李实）

*Mudanjiang Forestry Administrative Bureau of Heilongjiang Province, Mudanjiang 157000, P. R. China*

Cui Hong(崔红)

*Yichun Forestry School of Heilongjiang Province, Yichun 153000, P. R. China*

Li Changsheng(李长胜)   Liu Peng（刘鹏）  Zhang Junhua（张俊华）

*Northeast Forestry University, Harbin 150040, P. R. China*

**Abstract**   According to the growth characteristics of natural Korean pine (*Pinus Koraiensis*) forests, 6 equations such as Chapman-Richards equation, Logistic equation, Power equation, and so on were selected to fit for the growth models for Korean pine forest. The growth models were developed based on 208 random trees and 240 dominant trees. Results show that the Chapman-Richards equation is the best model for estimating tree height by age and DBH, while the Parabola equation is fittest for predicting DBH by age or estimating age from DBH. The site index table of Korean pine forest was compiled by using the proportional method with the Chapman-Richards equation as the guide curve and validated by accuracy test.
**Key words:** Korean pine forest, Growth model, Site index table

## Introduction

Korean pine (*Pinus koraiensis* Sieb. et Zucc.) is one of the most important timber species and it is naturally distributed in Heilongjiang, Jilin and Liaoning provinces in China. Many forestry researchers made extensive and profound studies on this species Jiang Yiyin (1985) studied the growth and growth models for plantations of Korean pine. However, very few papers were found on the growth models of natural Korean pine forest. Korean pine has a long growth period. For extensively managing Korean pine forest, it needs the forestry tables with high accuracy. In this study, the growth models and site index table of natural Korean pine forests were set up based on several equations and adequate samples of random trees and dominant trees.

## Methods

### Data collection

The studied site is located in Mudanjiang Region, Heilongjiang Province (128° 51'~134° 5' E and 43°40'~49° 25' N). Elevation is 400-800 m. The average annual temperature is from 2.5 °C to 4 °C. Frost-free period is 110~140 d. The mean annual precipitation is 460~690 mm. Data of 208 random trees and 240 dominant trees were collected from 240 temporary plots that are located in 10 forestry

bureaus of Mudanjiang Forest Administrative Bureau in Heilongjiang Province. The great number of plots distributed in wide-ranging and had samples representative of Korean pine forest Summary statistics for stands are presented in Table 1.

### Selection of equation

According to the growth characteristics of Korean pine natural forests and previous studies (Jiang 1990; Li 1988), the following equations, which are widely used as growth models by forestry scholars, were selected to fit the growth models of natural Korean pine forest.
Chapman-Richards equation

$$Y = a(1 - \exp(-bX)^c \qquad (1)$$

Mitscherlich equation

$$Y = a(1 - \exp(-cX)) \qquad (2)$$

Logistic equation:

$$Y = \frac{a}{1 - b\exp(-cX} \qquad (3)$$

Parabola equation:

$$Y = a + bX + cX^2 \qquad (4)$$

Power equation:

$$Y = aX^b \qquad (5)$$

Log equation:

$$Y = a + bX + cX^2 \tag{6}$$

Where in above equations, $Y$, $X$ are growth factors; $A$ is age, $a$, $b$, $c$ are parameters to be estimated.

The above 6 equations were extensively used on the development of growth models. The front 3 equations are theoretical growth equations. The Chapman-Richards equation is one of the growth equations with the flexible shape (Richards 1959). It can describe the growth affected by the many biological elements. And it's most applied both on the individual and stand models (Zeide 1993).

**Table 1 Attributes statistics of Stands form 240 plots**

| Stand variable | Minimum | Maximum | Mean | Standard deviation | Coefficient of the variation |
|---|---|---|---|---|---|
| Stand age (a.) | 25 | 190 | 89 | 33.689 | 0.379 |
| Mean DBH, cm | 8.0 | 90.0 | 24.3 | 9.585 | 0.395 |
| Mean height, m | 8.0 | 29.0 | 17.3 | 4.132 | 0.238 |
| Stand volume, $m^3 \cdot km^{-2}$ | 4 | 999 | 186.1 | 143.876 | 0.773 |

## Results and analysis

### Random trees

Using a program written with Marquart integration methods (Marquardt 1963) and data of 208 random trees, the above 6 equations were computed. Based on the standard of maximum of $R^2$ and the minimum of $S_{y \cdot x}$ (Jiang et al 1990; Lang et al. 1989), the optimal growth models for random trees are selected and presented in Table 2.

**Table 2. Optimal growth models for random trees**

| Factors | Growth models | $R^2$ | $S_{y \cdot x}$ |
|---|---|---|---|
| Age--height | $H=25.93048(1-\exp(-0.01682A))^{1.34705}$ | 0.828 | 1.790 |
| Age--DBH | $D= -6.48397+0.48943A-0.000833A^2$ | 0.800 | 3.251 |
| DBH--height | $H=26.66222(1-\exp(-0.04391D))^{1.05747}$ | 0.896 | 1.778 |
| DBH--age | $A=9.19462+3.396\ 09D-0.010358D^2$ | 0.877 | 5.124 |

Note: $H$, $D$ and $A$ in the table represent tree height, diameter and age respectively.

### Dominant trees

Using the data of 240 dominant trees, the above 6 equations were fitted. The optimal growth models are presented in Table 3.

**Table 3 Optimal growth models for dominant trees**

| Factors | Growth models | $R^2$ | $S_{y \cdot x}$ |
|---|---|---|---|
| Age--height | $H = 25.83103(1-\exp(-0.01883A))^{1.55851}$ | 0.889 | 1.771 |
| Age--DBH | $D = -7.97802 + 0.48954A-0.00078674A^2$ | 0.889 | 2.874 |
| DBH--height | $H = 26.81359(1-\exp(-0.04428D))^{1.02806}$ | 0.891 | 1.706 |
| DBH--age | $A = 8.25154 + 3.4926\ 0D-0.01264D^2$ | 0.877 | 5.124 |

Note: $H$, $D$ and $A$ in the table represent tree height, diameter and age respectively.

From Table 2 and 3, we see that the Chapman-Richards equation is the best model for predicting tree height from age and DBH. The Parabola equation is the best model for estimating DBH from age and estimating age from DBH. From the general tendency, the fitting result of dominant trees is better than that of random trees, as a result of that there existed some depressed trees among random trees, which are in older age but in small DBH and height.

## Site index table

From the above result, it shows that the Chapman-Richards equation is the best equation to pre-dict dominant height from age. The following equation is used for fitting the guide curve:

$$H_g = 25.83103[1-\exp(-0.01883A)]^{1.55851} \tag{7}$$

Where $H_g$ is height of dominant trees; $A$ is age.

By analyzing the growth data of stem analysis trees, the increment of most dominant trees of Korean pine was highest at age about 100. So 100 years was selected as the base age and the width of site index classes was set to 2 m. The proportional method (Clutter et al. 1983) was used to derive a set of height growth curves. The site index table of Korean pine is presented in Table 4.

In order to test the accuracy of the site index table, 54 dominant trees, which had not been fitted for constructing the site index table, were used to the accuracy test.

We estimated dominant height $H_\phi^*$ from age with equation (7). Then we fitted the following linear model:

$$H_\phi = a + bH_\phi^* \qquad (8)$$

and got: $a$=0.58679, $b$=1.01628, $R$=0.84, $n$=54, average relative error =2.15%. Then, calculated the following $F$-statistic:

$$F = \frac{(n - 2[na2 - 2a(b-1)\sum H_\phi^* - (b-1)\sum H_\phi^{*2}]}{2\sum(H_{\phi_1} - a - b\sum H_\phi^*)^2} \qquad (9)$$

Where $H_a$ is height of dominant trees; $H_\phi^*$ = estimated dominant height, $n$ is number of sample trees, $a$ and $b$ are the parameters of equation (8). We got: $F$=2.57148<$F_{54}$(0.05)=3.15421. The average height growth curve is therefore validated and the site index table is suitable for forestry practice.

Table 4. The site index table of Korean pine

| Age class | Dominant height /m | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 |
| 20 | 2.12 | 2.55 | 2.97 | 3.40 | 3.82 | 4.26 | 4.67 | 5.10 | 5.52 |
| 40 | 4.80 | 5.76 | 6.71 | 7.67 | 8.63 | 9.59 | 10.55 | 11.51 | 12.47 |
| 60 | 7.04 | 8.45 | 9.86 | 11.26 | 12.67 | 14.08 | 15.49 | 16.90 | 18.30 |
| 80 | 8.75 | 10.50 | 12.25 | 14.00 | 15.75 | 17.50 | 19.25 | 21.00 | 22.75 |
| 100 | 10.00 | 12.00 | 14.00 | 16.00 | 18.00 | 20.00 | 20.00 | 24.00 | 26.00 |
| 120 | 10.89 | 13.07 | 15.25 | 17.43 | 19.60 | 21.78 | 23.96 | 26.14 | 28.32 |
| 140 | 11.52 | 13.82 | 16.13 | 18.43 | 20.73 | 23.04 | 25.34 | 27.64 | 29.95 |
| 160 | 11.96 | 14.35 | 16.74 | 19.13 | 21.52 | 23.91 | 26.30 | 28.69 | 31.09 |
| 180 | 12.26 | 14.71 | 17.16 | 19.62 | 22.07 | 24.52 | 26.97 | 29.42 | 31.88 |
| 200 | 12.47 | 14.96 | 17.46 | 19.95 | 22.45 | 24.94 | 27.43 | 29.93 | 32.42 |
| 220 | 12.61 | 15.14 | 17.66 | 20.18 | 22.71 | 25.23 | 27.75 | 30.27 | 32.80 |
| 240 | 12.71 | 15.26 | 17.80 | 20.34 | 22.89 | 25.43 | 27.97 | 30.51 | 33.68 |
| 260 | 12.78 | 15.34 | 17.90 | 20.45 | 23.01 | 25.57 | 28.12 | 30.68 | 33.23 |
| 280 | 12.83 | 15.40 | 17.96 | 20.53 | 23.09 | 25.66 | 28.23 | 30.79 | 33.36 |

## Conclusion

In this study, growth models for natural Korean pine forests were developed. The Chapman-Richards equation is the best model for predicting height from age and DBH. The Parabola equation is the best model for estimating DBH from age and estimating age from DBH. When the site index table of natural Korean pine forests was tested by the data that had not been used for constructing the table, its accuracy is qualified and can be directly put into use of forestry practice.

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